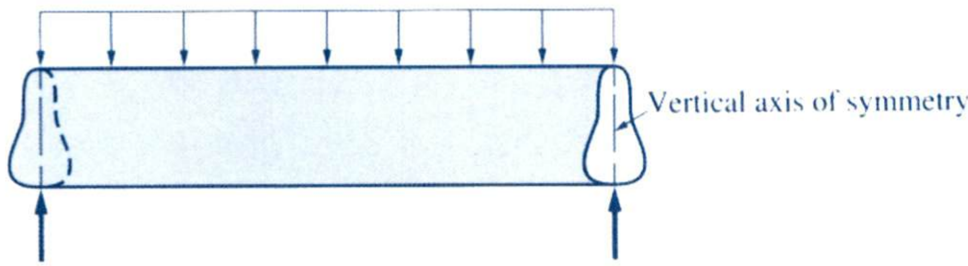
Chapter 13 - Shear Forces and Bending Moments in Beams
Reading: Chapter 13 — Pages 449-475

13-1
Introduction

- Many structures can be approximated as a straight beam or as a collection of straight beams. For this reason, the analysis of stresses and deflections in a beam is an important and useful topic.
- This section covers shear force and bending moment in beams, shear and moment diagrams, stresses in beams, and a table of common beam deflection formulas.

Beam Assumptions

1. Straight and of uniform cross-section, and that possess a vertical plane of symmetry, as shown below.
2. Horizontal, although in actual situations beams may be inclined or in vertical positions.
3. Subjected to forces applied in the vertical plane of symmetry, as shown below.

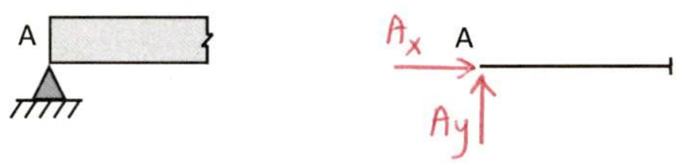


13-2
Types of Beams

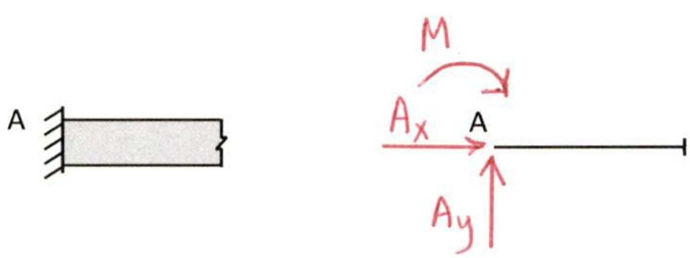
Types of Beam Support
Three main types of supports and Reactions

Roller
One Unknown Reaction

Pin (Hinge)
Two Unknown Reactions
(x & y - components)



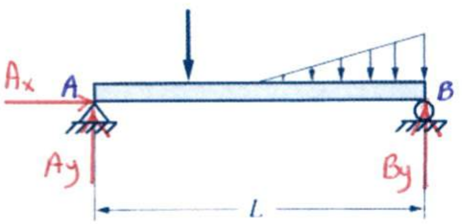
Fixed
Three unknown Reactions
▷ Translation (x & y)
▷ Rotation (Moment)



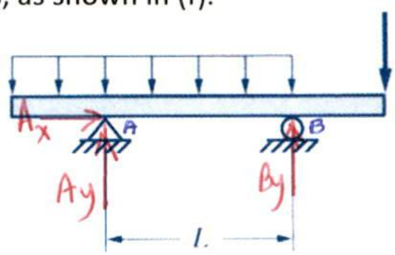
Types of Beams

Beams can be classified into the types shown below, according to the kind of support used.

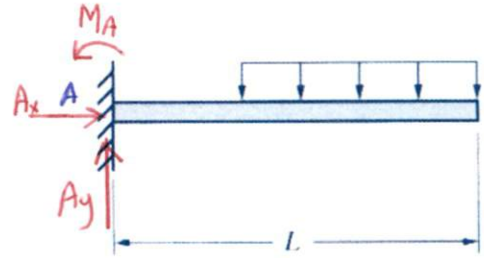
Simple Beam	A beam supported at its ends with a hinge and a roller, as shown in (a), is called a simple beam.
Overhanging Beam	A simply supported beam with an overhang from one or both ends, as shown in (b) is called an overhanging beam.
Cantilever Beam	A beam that is fixed at one end and free at the other, as shown in (c), is called a cantilever beam.
Propped Cantilever Beam	A beam that is fixed at one end and simply supported at the other, as shown in (d), is called a propped cantilever beam.
Fixed Beam	When both ends of a beam are fixed to supports, as shown in (e), the beam is called a fixed beam.
Continuous Beam	A continuous beam is supported on a hinge support and two or more roller supports, as shown in (f).



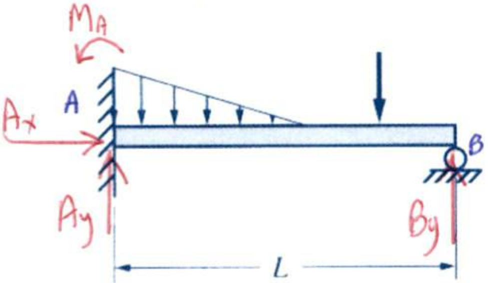
(a) Simple beam



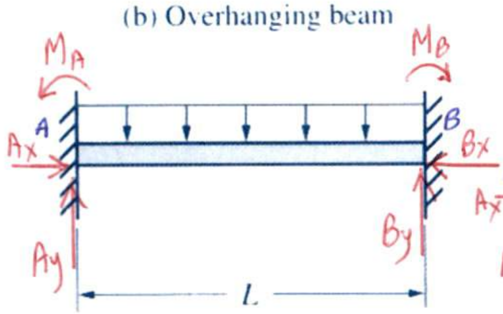
(b) Overhanging beam



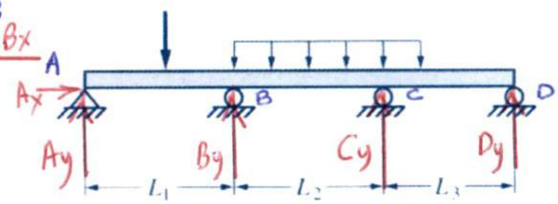
(c) Cantilever beam



(d) Propped cantilever beam



(e) Fixed beam



(f) Continuous beam

Conditions of Equilibrium:

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M_A = 0$ (about any point)

The three equations can be used to solve for no more than three unknowns.

Statically Determinate Beams. In the first three types of beams, shown in a, b, and c, there are three unknown reaction components that may be determined from the static equilibrium equations. Such beams are said to be statically determinate.

Statically Indeterminate Beams. When the number of unknown reaction components exceeds three, as in the beams shown in d, e, and f, the three equilibrium equations are insufficient for determining the unknown reaction components. Such beams are said to be statically indeterminate.

13-3
Types of Loading

Beams are subjected to various loads. Only the concentrated, uniform, and linearly varying loads will be discussed here.

Concentrated Loads

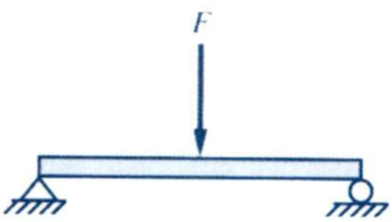
A concentrated load is applied at a specific point on the beam and is considered as a discrete force acting at the point, as shown in (a). For example, a weight fastened to a beam by a cable applies a concentrated load to the beam.

Uniform Loads

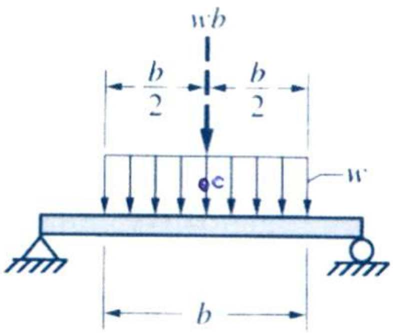
When a load is distributed over a part or the entire length of the beam, it is called a distributed load. If the intensity of a distributed load is a constant value, it is called a uniform load. The load intensity is expressed as force per unit length of the beam, such as lb/ft or N/m. For computing the reactions, the distributed load may be replaced by its equivalent force. The equivalent force of a uniform load is equal to the load intensity w multiplied by the length of distribution b, and the line of action of the equivalent force passes through the midpoint of the length b, as shown in (b). The weight of a beam is an example of a uniformly distributed load.

Linearly Varying Loads

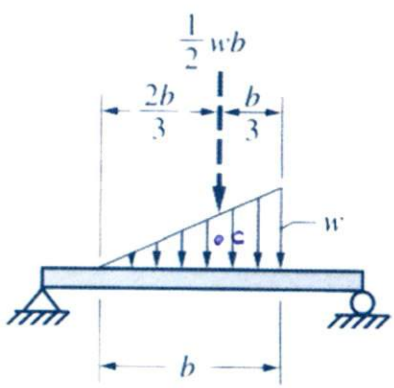
A linearly varying load is a distributed load with a uniform variation of intensity. Such a load condition occurs on a vertical or inclined wall due to liquid pressure. Example (c) shows a linearly varying load, with intensity varying linearly from zero to a maximum value w.



(a) Concentrated load



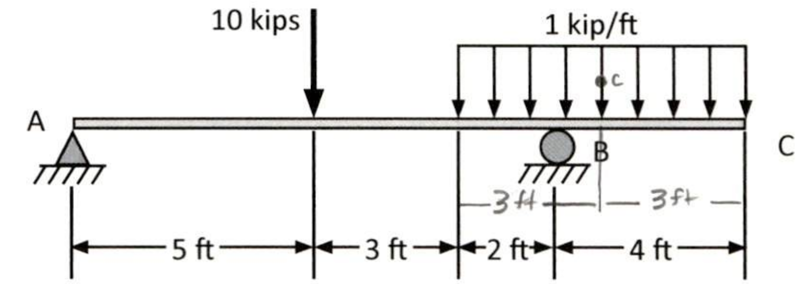
(b) Uniform load



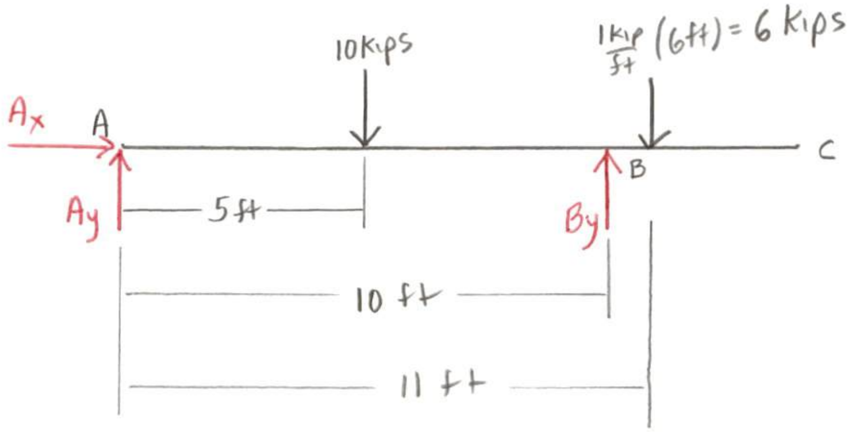
(c) Linearly varying load

13-4
Beam Reactions

Example 13-1
Determine the external reactions at the supports A and B for the overhanging beam due to the loading shown.



Solution.



FBD

ccw +M
cw −M

Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad A_x = 0$

$[\Sigma M_A = 0]$ $\quad -10\text{ kips}(5\text{ ft}) + B_y(10\text{ ft}) - 6\text{ kips}(11\text{ ft}) = 0$

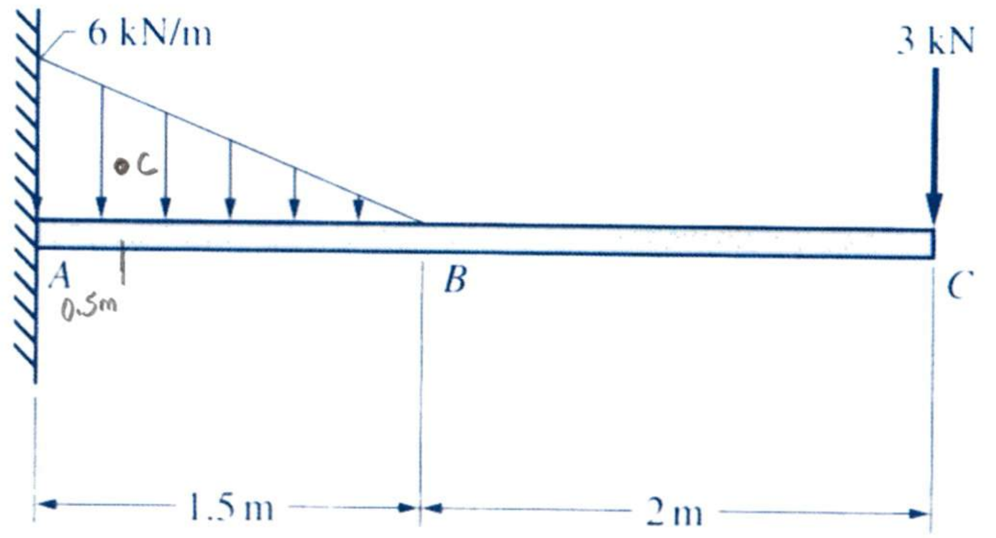
$$B_y = \frac{116\text{ kips}\cdot\text{ft}}{10\text{ ft}} = \underline{\underline{11.6\text{ kips}}}\ \uparrow$$

$[\Sigma F_y = 0]$ $\quad A_y - 10\text{ kips} + B_y - 6\text{ kips} = 0$

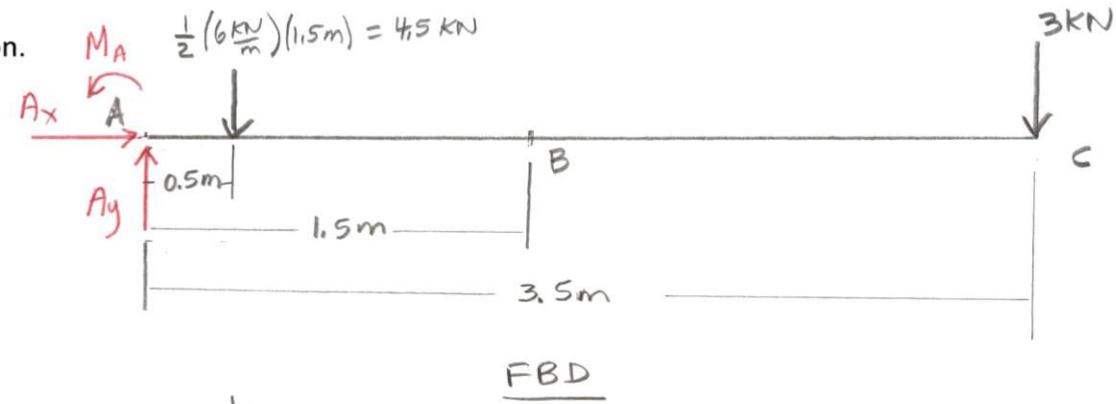
$$A_y = 16\text{ kips} - 11.6\text{ kips} = \underline{\underline{4.4\text{ kips}}}\ \uparrow$$

Example 13-2

Determine the external reactions at the fixed support of the catilever beam due to the loading shown.



Solution.



Equilibrium Equations

CCW +M

CW −M

$[\Sigma F_x = 0]$ $A_x = 0$

$[\Sigma F_y = 0]$ $A_y - 4.5kN - 3kN = 0$

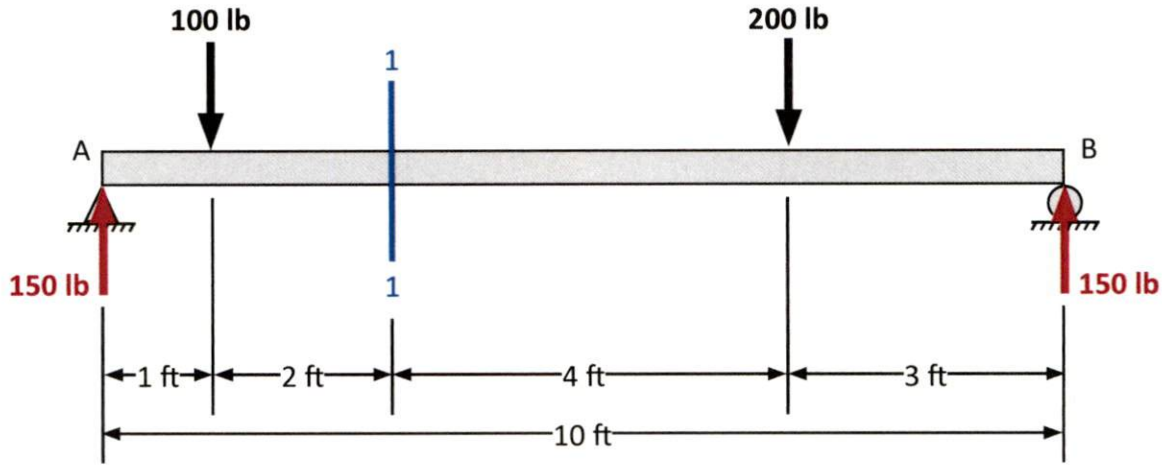
$A_y = \underline{\underline{7.5\ kN}} \uparrow$

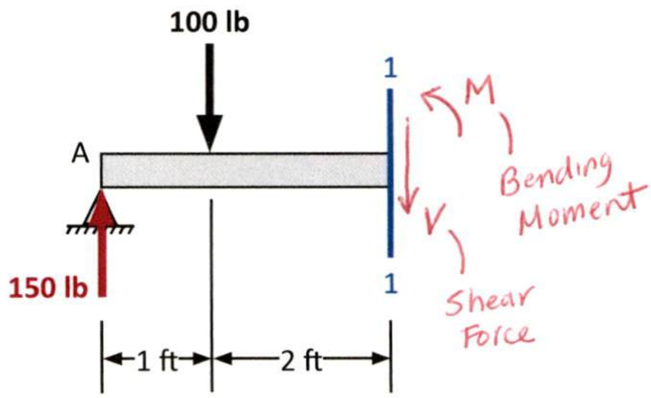
$[\Sigma M_A = 0]$ $M_A - 4.5kN(0.5m) - 3kN(3.5m) = 0$

$M_A = \underline{\underline{12.75\ kN \cdot m}}$

13-5
Shear Force and Bending Moment in Beams

Internal shear force and bending moment are developed in a beam to resist the external forces and to maintain equilibrium.





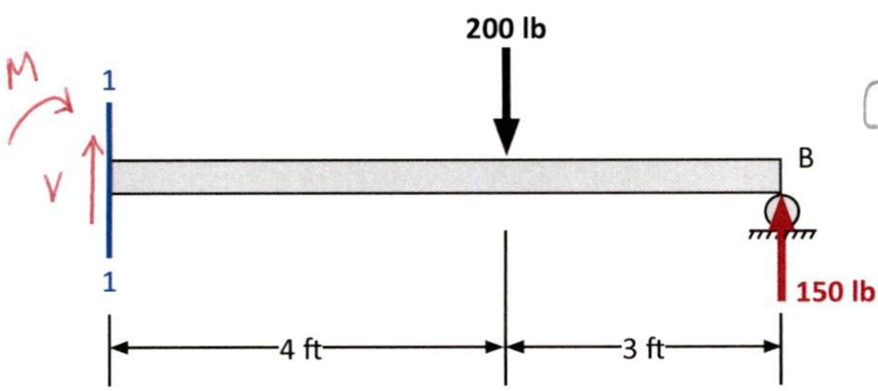
FBD - Left Portion of Section 1-1

$[\Sigma F_y = 0]$ $150\,lb - 100\,lb - V = 0$

$V = 50\,lb \downarrow$

$[\Sigma M_{1\text{-}1} = 0]$ $-150\,lb(3\,ft) + 100\,lb(2\,ft) + M = 0$

$M = 250\,lb \cdot ft$



FBD - Right Portion of Section 1-1

$[\Sigma F_y = 0]$

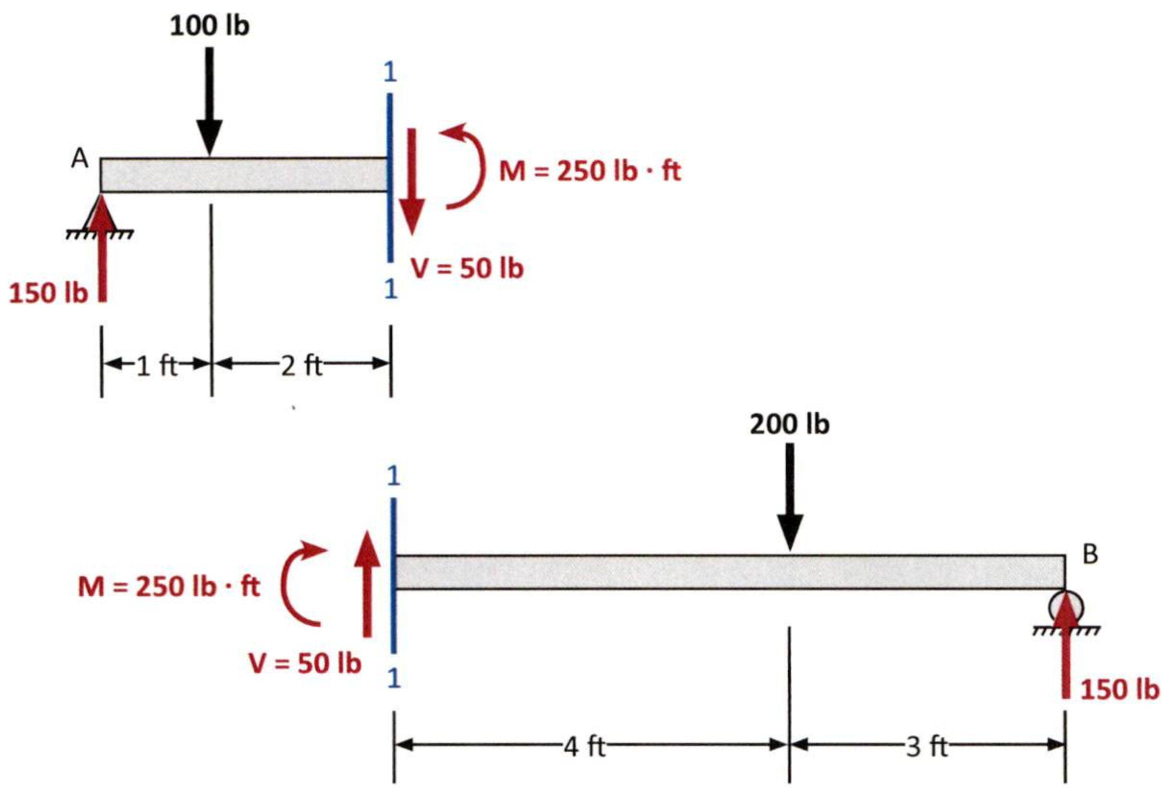
$V - 200\,lb + 150\,lb = 0$

$V = 50\,lb \uparrow$

$[\Sigma M_{1\text{-}1} = 0]$

$150\,lb(7\,ft) - 200\,lb(4\,ft) - M = 0$

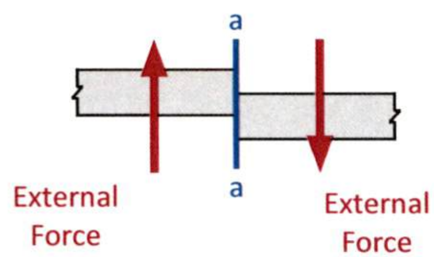
$M = 250\,lb \cdot ft$



Beam Sign Conventions

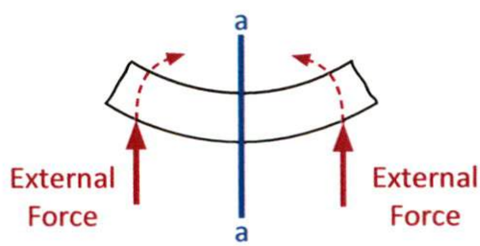
Signs for the internal shear forces and bending moments are based on the effects that they produce:

1. Positive Shear. The shear force at a section is positive if the external forces on the beam produce a shear effect that tends to cause the left side of the section to move up relative to the right side.

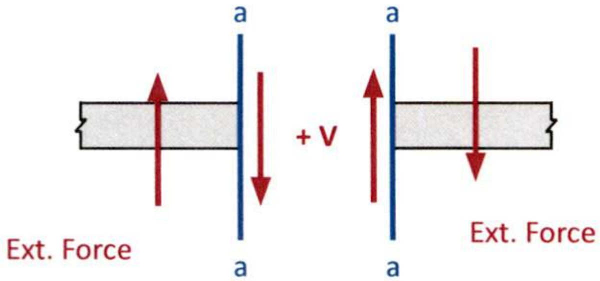


Effect of Positive Shear

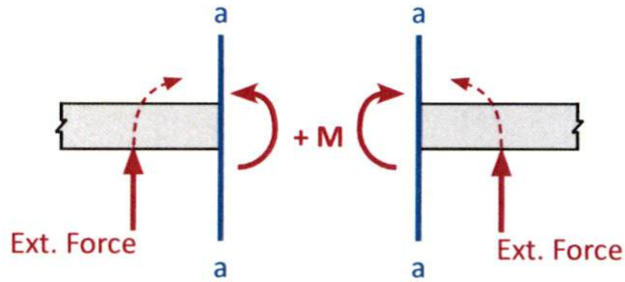
2. Positive Moment. The bending moment of a section is positive if the external forces on the beam produce a bending effect that causes the beam to bend concave upward (the center of curvature is above the curve) at the section.



Effect of Positive Moment



Direction of Positive Internal Shear Force V



Direction of Positive Internal Bending Moment M

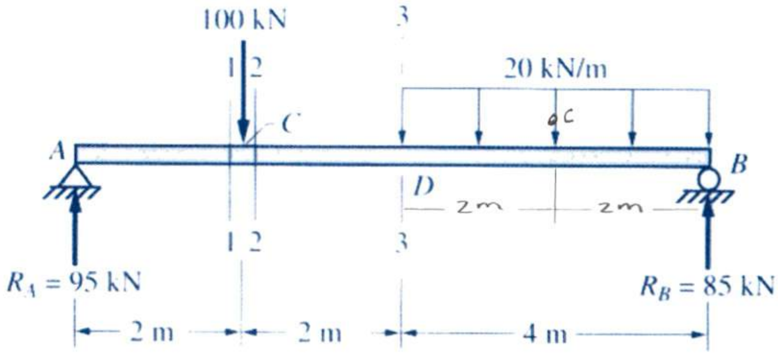
Rule 1 **(For Finding Shear Forces)** The internal shear force at any section of a beam is equal to the algebraic sum of the external forces on either segment separated by the section. If the summation is from the left end of the beam to the section, treat the upward forces as positive. If the summation is from the right end of the beam to the section, treat the downward forces as positive.

$$V = \Sigma \text{ Ext. Forces} \begin{cases} \text{From Left:} & \text{Upward force as positive} \\ \text{From Right:} & \text{Downward force as positive} \end{cases}$$

Rule 2 **(For finding Bending Moment)** The internal bending moment at any section of a beam is equal to the algebraic sum of the moments about the section due to the external forces on either segment separated by the section, In either case, treat the moment produced by upward forces as positive.

$$M = \Sigma \text{ Moments of Ext. Forces} \begin{cases} \text{From either side:} \quad \text{Moment due to} \\ \text{upward force as positive} \end{cases}$$

Example 13-3 Calculate the shear forces and bending moments at sections C and D of the beam shown.



(Rule#1)

Solution. Shear force at C

The shear force right at the section where a concentrated force is applied is undefined.

∴ We must choose a section a little to the left (section 1-1) and another section a little to the right of C (section 2-2).

$V_{C^-} = +95\,kN$

$V_{C^+} = +95\,kN - 100\,kN = -5\,kN$

Shear Force at D

Section 3-3 at D

(Ext. Forces from A to D) $V_D = V_{3-3} = +95\,kN - 100\,kN = -5\,kN$

SAME!

(Ext. Forces from B to D) $V_D = V_{3-3} = -85\,kN + 20\frac{kN}{m}(4m) = -5\,kN$

(Rule #2)

Bending Moment

Starting from the left end of the Beam:

$$M_C = +95\,kN(2m) = 190\ kN\cdot m$$

$$M_D = +95\,kN(4m) - 100\,kN(2m) = 180\ kN\cdot m$$

Starting from the right end of the Beam:

$$M_C = +85\,kN(6m) - 80\,kN(4m) = 190\ kN\cdot m$$

$$M_D = +85\,kN(4m) - 80\,kN(2m) = 180\ kN\cdot m$$